QML: A functional quantum programming language

quantum control and orthogonality

Jonathan Grattage
with Thorsten Altenkirch & Alex Green

sneezy.cs.nott.ac.uk/qml
School of Computer Science & IT,
The University of Nottingham

What is QML?

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- Project Site: QML@CS.Nott – sneezy.cs.nott.ac.uk/qml

Quantum Languages

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- *Quantum* data, *Classical* control

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- Notion of Finite Quantum Computations (**FQC**) developed by analogy with Finite Classical Computations (**FCC**)

Classical v. Quantum

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Classical v. Quantum

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Projections	Partial trace

QML Syntax

- Types:

$$\sigma = \mathcal{Q}_{\mathbf{1}} \mid \mathcal{Q}_{\mathbf{2}} \mid \sigma \otimes \tau$$

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- Syntax:

$$\begin{array}{lll}
(\mathit{Variables}) & x, y, \ldots \in & \mathit{Vars} \\
(\mathit{Prob.amplitudes}) & \kappa, \iota, \ldots \in & \mathbb{C} \\
(\mathit{Patterns}) & p, q & ::= x \mid (x, y) \\
(\mathit{Terms}) & t, u, e & ::= x \mid x^{\vec{y}} \mid () \mid (t, u) \\
 & & \mid \quad \mathbf{let}\ p = t\ \mathbf{in}\ u \\
 & & \mid \quad \mathbf{if}\ \ t\ \mathbf{then}\ u\ \mathbf{else}\ u' \\
 & & \mid \quad \mathbf{if}^{\circ}\ t\ \mathbf{then}\ u\ \mathbf{else}\ u' \\
 & & \mid \quad \mathit{false} \mid \mathit{true} \mid \overrightarrow{0} \mid \kappa \times t \mid t + u
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- EPR State: $\frac{1}{\sqrt{2}} \times (\mathit{false}, \mathit{false}) + \frac{1}{\sqrt{2}} \times (\mathit{true}, \mathit{true})$

Control of Decoherence

- Projection Function

$$\pi_1 \in (\mathcal{Q}_{\mathbf{2}}, \mathcal{Q}_{\mathbf{2}}) \rightarrow \mathbf{C}_{\mathbf{2}}$$

$$\pi_1\ (x, y) = x$$

$$Q_2 \longrightarrow Q_2$$

$$Q_2 \longmapsto$$

$$\phi_{\pi_1}$$

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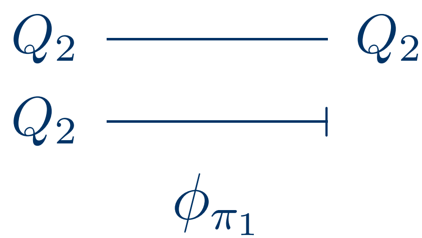
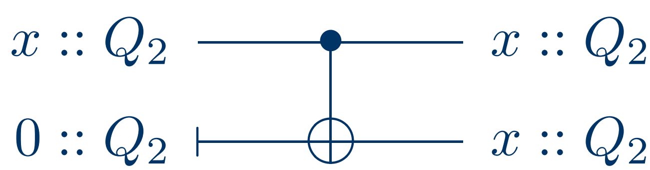


- Diagonal Function

$$\delta \in \mathcal{Q}_{\mathbf{2}} \longrightarrow (\mathcal{Q}_{\mathbf{2}}, \mathcal{Q}_{\mathbf{2}})$$
$$\delta\ x = (x, x)$$



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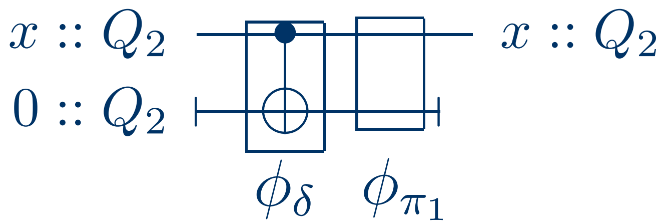
- $\pi_1.\delta \in \mathcal{Q}_{\mathbf{2}} \to \mathcal{Q}_{\mathbf{2}}$



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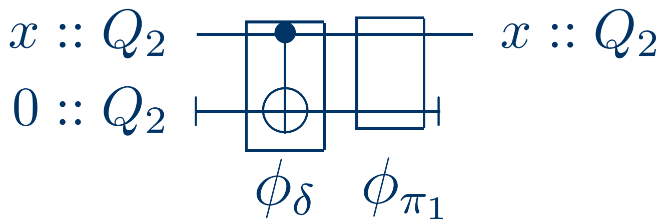

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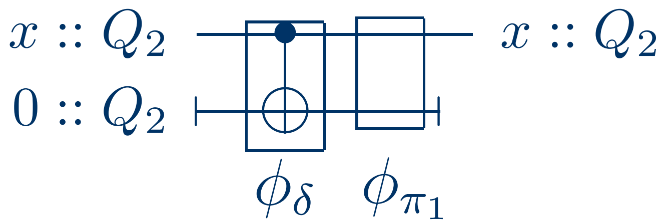

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$$Q_2 \longrightarrow Q_2$$

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  Input $= \frac{1}{\sqrt{2}} \times \mathit{false} + \frac{1}{\sqrt{2}} \times \mathit{true}$ (equal superposition)

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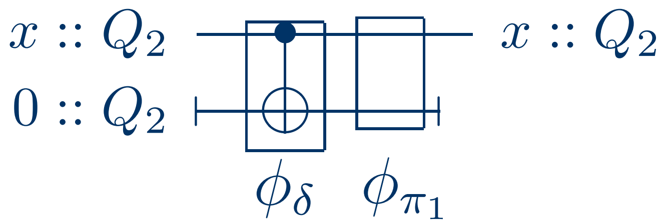

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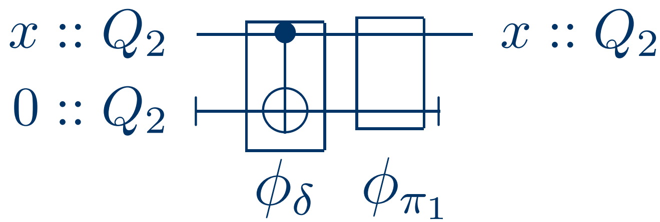

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  Decoherence! Not the identity function

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- Not, using **if**

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- $$cnot\ c\ x = \mathbf{if}^\circ\ c\ \mathbf{then}\ (true, qnot\ x)\ \mathbf{else}\ (false, x)$$

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$$\mathit{cnot}\ c\ x = \mathbf{if}^{\circ}\ c\ \mathbf{then}\ (\mathit{true}, \mathit{qnot}\ x)\ \mathbf{else}\ (\mathit{false}, x)$$
- Deutsch-Joza Algorithm, Quantum Teleport Algorithm, ...

Quantum Teleport Algorithm

$$pZed \in \mathcal{Q}_\mathbf{2} \multimap \mathcal{Q}_\mathbf{2}$$

$$pZed\ x = \mathbf{if}^\circ\ x\ \mathbf{then}\ (-1) \times \mathit{true}\ \mathbf{else}\ \mathit{false}$$

$$had \in \mathcal{Q}_\mathbf{2} \multimap \mathcal{Q}_\mathbf{2}$$

$$had\ x = \mathbf{if}^\circ\ x\ \mathbf{then}\ (-1) \times \mathit{true} + \mathit{false}\ \mathbf{else}\ \mathit{true} + \mathit{false}$$

$$tel \in \mathcal{Q}_\mathbf{2} \multimap \mathcal{Q}_\mathbf{2}$$

$$\begin{aligned} tel\ x = \ \mathbf{let}\ (a, b) &= (\mathit{false}, \mathit{false}) + (\mathit{true}, \mathit{true}) \\ (a', x') &= cnot\ a\ x \\ b' &= \mathbf{if}\ a'\ \mathbf{then}\ qnot\ b\ \mathbf{else}\ b \\ b'' &= \mathbf{if}^\circ\ had\ x'\ \mathbf{then}\ pZed\ b'\ \mathbf{else}\ b' \\ \mathbf{in}\ b'' & \end{aligned}$$

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- Other work:
- A compiler for QML, in Haskell, that produces (typed) circuits

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Summary

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- Extend algebra and normal form beyond pure subset of QML
- Higher-order types, composionality proofs, implement using measurement calculus ...

Thanks for listening

- Papers on QML can be found at:
- sneezy.cs.nott.ac.uk/qml
- There is also an interactive research diary:
- sneezy.cs.nott.ac.uk/qml/weblog

- Jonathan Grattage (www.cs.nott.ac.uk/∼ jjg)
  Thorsten Altenkirch (www.cs.nott.ac.uk/∼ txa)